DETERMINING THE RADIUS OF AN AIR VORTEX IN A CENTRIFUGAL EJECTOR

M. A. Gol'dshtik, G. P. Zykin, Yu. I. Petukhov, and V. N. Sorokin

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 4, pp. 107–111, 1969

The maximum-discharge principle is widely used to calculate the radius of the air vortex [1]. However, this principle has not been justified theoretically. Other extremal principles ([2], for example) which yield similar results have also been proposed.

Several attempts have been made to justify the maximum-discharge principle [3, 4]. However, these studies have essentially shown that this principle admits various equivalent formulations. Here it is important to note that the maximum-discharge principle was formulated for weirs with a broad threshold, a particular case being the centrifugal ejector with a long nozzle. Therefore this principle applies ideally to the case of an infinite nozzle [1]. Our analysis is made under the assumption that the nozzle is infinitely long (phenomena at the nozzle exit section are not considered).

Within the framework of the hydraulic formulation of the problem, which has been the only approach used to date, the use of extremal principles or other additional conditions is necessary. The objective of the present study is the hydrodynamic solution of the problem of ideal fluid flow in a geometry which differs in the aforementioned way from the geometry of the real centrifugal ejector. In this formulation we have a typical problem of ideal fluid jet theory, whose solution does not require additional conditions of the extremal principle type. The condition that the ideal fluid flow be potential supplies the missing relation for closure of the system of equations in the hydraulic formulation. The hydrodynamic methods for solution of the problem make it possible to determine the physical meaning of the maximum-discharge principle.

1. Numerical solution of the problem. Consider axisymmetric flow of an ideal incompressible fluid in the region shown in Fig. 1. Here DCC'D' is a segment of the infinite nozzle. In the calculations the section CC' is considered to be sufficiently distant from the entrance so that the flow at this section can be considered uniform; ABB'A' is the liquid free surface. The swirling fluid stream enters through FEE'F'. The overall fluid discharge rate Q and the velocity circulation Γ are given.

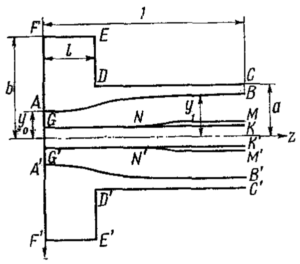

Fig. 1. Axial section of idealized centrifugal ejector.

We introduce r, z cylindrical coordinates and seek the harmonic velocity potential function determined from the following conditions:

$$\frac{\partial\varphi}{\partial n}=0 \quad \text{on } AB, CD, DE, FA; \qquad \frac{\partial\varphi}{\partial n}=-1 \quad \text{on } EF$$

$$\frac{\partial\varphi}{\partial n}=\frac{2bl}{a^2-y_1^2} \quad \text{on } BC, \qquad \Phi=v^2+\frac{v_0^2b^2}{y^2}-\frac{v_0^2b^2}{y_0^2}=0 \quad \text{on } AB \tag{1.1}$$

$$(v_0=\Gamma/2\pi b, \quad v^2=v_r^2+v_z^2)$$

Here $r = y(z)$ is the equation of the unknown-free-surface generatrix. The last condition requires that the pressure on surface AB be constant. In all the preceding formulas the discharge Q was taken as the scale for the dimensionless potential, and the radial velocity at the section FEE'F' was taken as the velocity scale.

For later use we find from (1.1)

$$\frac{1}{y_0^2} = \frac{1}{y_1^2} + \frac{4l^2}{v_0^2 (a^2 - y_1^2)^2} \tag{1.2}$$

We assume that curve AB is given. Then the function φ is found by solving the Neumann problem

$$\Delta\varphi = 0, \qquad \partial\varphi / \partial n = f(N)$$

Here n is the outward normal to the boundary of the region, and N is a point on the boundary. By use of the Green's formula this problem can be reduced to an integral equation of the form

$$\int_S [\varphi(N) - \varphi(N_0)]\, r \frac{\partial G}{\partial n}\, dS = \int_S f(N)\, rG dS \tag{1.3}$$

Here

$$G = \frac{x}{\sqrt{2q}} K(x), \qquad x = \left(\frac{2q}{p+q}\right)^{1/2},$$

$$q = 2rr_0, \qquad p + q = (r - r_0)^2 + (z - z_0)^2$$

Here N_0 is the "observation point"; S is the contour bounding the flow region; K(x) is the complete elliptic integral of the first kind.

For the numerical solution the integrals were replaced by finite sums using the Gauss quadrature formulas. Eight points were taken on all the segments other than BC and EF; on BC and EF one point was taken at the midpoint of each interval. The singularity on the right-hand side of (1.3) was isolated on the basis of the asymptotic representation

$$K(x) \sim \ln \frac{4}{\sqrt{1 - x^2}} + O(1 - x^2) \text{ when } x \to 1$$

Denoting the selected points by N_i ($i = 1, \ldots, 34$), from (1.3) we obtain the system of linear equations

$$\sum c_k (\varphi_k - \varphi_i)\, r_k \frac{\partial G}{\partial n}\Big|_{k,i} = \Phi_i \qquad (i = , \ldots, 34)$$

where Φ_i is the value of the right-hand side, calculated for N_0N_i.

Since for k = i the corresponding term vanishes, we can write

$$\sum_{k=1}^{34} B_{k,i}\varphi_k = \Phi_i$$

$$B_{k,i} = c_k r_k \frac{\partial G}{\partial n}\Big|_{k,i} (k \neq i), \qquad B_{k,i} = -\sum_{j \neq i} c_j r_j \frac{\partial G}{\partial n}\Big|_{j,i} \quad (k = i) \tag{1.4}$$

The velocity potential is determined to within a constant, and therefore we can set $\varphi_{34} = 0$ and drop one equation.

The final form of the system from which φ_i was found is

$$\sum_{k=1}^{33} B_{k,i}\varphi_k = \Phi_i \qquad (i = 1, \ldots, 33) \tag{1.5}$$

For a given boundary shape 38 seconds of machine time is required on the M-20 computer to find φ_i.

The velocity was calculated using the formula

$$v_i = \frac{1}{\sqrt{1 + r_i'^2}} \frac{\partial \varphi_i}{\partial z}$$

where the differentiation was performed numerically using finite increments. This scheme makes it possible to solve the problem when the shape of the flow region is known.

To find the unknown free boundary, curve AB was varied until condition (1.1) was satisfied at the eight interior points. The unknowns were the ten ordinates of the free boundary—eight interior and two end point ordinates. The two additional conditions were relation (1.2) and the condition $dy/dz = 0$ for $z = 0$, expressing the requirement that the free surface be orthogonal to the end wall AF. The resulting nonlinear system of equations was solved by a method consisting of a combination of the Newton-Kolmogorov method and a modification thereof.

The magnitude of the relative radius y_1/a of the air vortex found in this way is shown as curve 2 in Fig. 2. The abscissa is the quantity $m = 2Q/\Gamma a = 1/A$, where A is the geometric characteristic of the ejector.

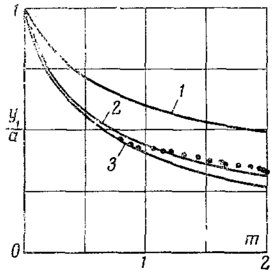

Fig. 2. Asymptotic radius of the annular discontinuity versus the parameter m.

2. **Approximate problem solution.** We take the outer radius R of the exit tube as the length scale.

We consider ideal fluid flow in a region similar to the flow region in the ejector (Fig. 1). Here GG'K'K is the inner cylindrical wall of radius r_0. For sufficiently large values of A, when the point of liquid separation lies on FF', this wall does not affect the form of the region occupied by the liquid. With reduction of A, y_0 diminishes. When $y_0 = r_0$ the free surface contacts the inner cylinder at one point. Further reduction of A shifts the point of liquid separation from the wall FF' to GG'K'K (point N), and the coordinate Z_N of the point N is the larger the closer the value of A to the critical value A_0, for which the entire region between the two coaxial cylinders is filled. As $Z_N \to \infty$, $y_1 \to r_0$.

Let Z_N be large. In this case the influence of the liquid entrance conditions on the form of the free surface is vanishingly small and we can assume that flow takes place in an infinite strip as shown in Fig. 3. The larger Z_N, the more precisely the correspondence between the original flow (Fig. 1) and the flow in the strip (Fig. 3) is satisfied.

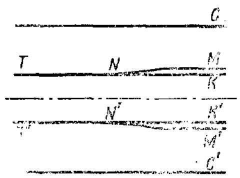

Fig. 3. Axial section of simplified flow region.

We introduce the stream function ψ, which is known to satisfy the conditions

$$\frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial r^2} - \frac{1}{r}\frac{\partial\psi}{\partial r} = 0, \qquad v_r = -\frac{1}{r}\frac{\partial\psi}{\partial z}, \qquad v_z = \frac{1}{r}\frac{\partial\psi}{\partial r} \tag{2.1}$$

Let $\psi = 0$ for $r = 1$ and $\psi = Q/2\pi$ on the line TNM. These are the standard boundary conditions for the stream function. Moreover, the constant pressure condition must be satisfied on the free boundary NM:

$$\left|\frac{1}{r}\frac{\partial\psi}{\partial r}\right|^2 + \frac{\Gamma^2}{4\pi^2 y^2} = \frac{\Gamma^2}{4\pi^2 r_0^2} + v_N^2 \tag{2.2}$$

where v_N is the unknown axial velocity at the separation point N. Let the curve TNM be described by the function $r = y(z)$. Then from (2.1), on NM with the aid of Lavrent'ev-Moiseev narrow strip theory [5, 6], which makes it possible to find the asymptotic Laplace equations in regions near the strip, we obtain

$$\frac{1}{y^2}\left(\frac{\partial\psi}{\partial r}\right)^2 = \frac{Q^2}{\pi^2(1-y^2)^2}\left\{1+y''\frac{y}{y^2-1}\left(\frac{y^2-1}{2}-1+\frac{\ln y^2}{y^2-1}\right)\right\} \tag{2.3}$$

In (2.3) terms proportional to the third and higher derivatives of y, which, as will be shown later, approach zero as $z_N \to \infty$, are dropped.

From (2.2) with account for (2.3) we obtain

$$\frac{1}{(1-\tau\delta^2)^2}\left\{1+\tau\frac{d^2\delta}{dz^2}\frac{\delta}{\tau\delta^2-1}\left(\frac{\tau\delta^2-1}{2}-1+\frac{\ln\tau\delta^2}{\tau\delta^2-1}\right)\right\} = \mu\frac{\delta^2-1}{\delta^2}+\nu \tag{2.4}$$

Here

$$\tau = r_0^2, \quad \delta = \frac{y}{r_0}, \quad \mu = \frac{\Gamma^2}{4r_0^2Q^2}, \quad \nu = v_N^2\frac{\pi^2}{Q^2}$$

From the definition of δ and the requirement that the free surface leave the wall along the tangent, we have at the point N the boundary conditions: $\delta = 1$, $d\delta/dz = 0$. Expanding the logarithm in (2.4) into a series and introducing the small quantity $s = \delta^2 - 1$, we obtain

$$\begin{aligned}\frac{\tau}{6}\frac{d^2s}{dz^2} &= \left[\frac{1}{1-\tau}-\nu(1-\tau)\right]+\left[\frac{\tau}{(1-\tau)^2}+\nu\tau-\mu(1-\tau)\right]s \\ &+\left[\mu(1-\tau)+\tau\mu+\frac{\tau^2}{(1-\tau)^3}\right]s^2\end{aligned} \tag{2.5}$$

Multiplying (2.5) by ds/dz and integrating (at the separation point $s = 0$ and $ds/dz = 0$), we obtain

$$\begin{aligned}\frac{\tau}{12}\left(\frac{ds}{dz}\right)^2 &= \left[\frac{1}{1-\tau}-\nu(1-\tau)\right]s+\left[\frac{\tau}{(1-\tau)^2}+\nu\tau-\mu(1-\tau)\right]\frac{s^2}{2}+ \\ &+\left[\mu(1-\tau)+\tau\mu+\frac{\tau^2}{(1-\tau)^3}\right]\frac{s^3}{3}\end{aligned} \tag{2.6}$$

For large but finite z_N and as $z \to \infty$, s approaches its asymptotic value s_0 and ds/dz approaches zero (asymptotically uniform flow). Hence it is clear that the right-hand side of (2.6) must have the multiple root S_0.

As $z_N \to \infty$

$$v_N^2 \to \frac{Q^2}{\pi^2(1-\tau)^2}, \qquad \nu \to \frac{1}{(1-\tau)^2}$$

therefore the root multiplicity condition leads to the filling criterion

$$\mu = \frac{2r_0^2}{(1-r_0^2)^3} \tag{2.7}$$

It is obvious that (2.7) coincides with the corresponding relation of the maximum-discharge principle but has a different meaning.

From the multiplicity condition for $\nu = 0$ we obtain the connection between the parameters for flow without the inner wall:

$$\mu = \frac{(2\tau+{}^{16}/_3)+\sqrt{(2\tau+{}^{16}/_3)^2+{}^{52}/_3\tau^2}}{2(1-\tau)^3} \qquad (\tau = y_0^2) \tag{2.8}$$

After differentiating (2.5) the required number of times, we can see that as $z_N \to \infty$, ds/dz, $d^2s/dz^2 \gg d^3s/dz^3$, d^4s/dz^4, and so on, and the inequality becomes stronger with increase of z_N, so that in the limit Eq. (2.3) becomes exact, and consequently the criterion (2.7) is exact.

Thus, the relations which result from the maximum-discharge principle have the meaning of the criterion for filling of the canonical region in question.

3. Experimental studies of axial annular flow parameters. The formulas of sections 1 and 2 were derived for an ideal liquid. Therefore in preparing for and conducting the experiment particular attention was devoted to reducing as far as possible the viscosity of the real liquid. This is achieved by high liquid velocities (up to 20 m/sec) and large characteristic dimensions of the flow region.

The chamber model is a cylinder composed of 32 vanes with a discharge nozzle. The vane axes were located on a 280-mm-diameter circle. The cylinder height was 30 mm. The chamber was located in a housing made in spiral form. The spiral inlet makes possible uniform distribution of the fluid around the circumference of the guide vanes. Each 90-mm-long vane can rotate about its axis through angles from 0 to 65° relative to the radius. Simultaneous rotation of all 32 vanes is accomplished with the aid of a special pivoting device. The inner diameter of the nozzle is 72 mm, and its length is 700 mm.

The most important characteristic of this flow geometry is the ratio of the liquid circulation (characterizing the intensity of the tangential motion) to the discharge rate. The described guide vane apparatus made it possible to change this ratio.

The inclination α of the vanes to the radius passing through the vane axis determines the numerical value of the ratio as follows. Let the liquid velocity in the space between the vanes be v_0. Then at the radius R

$$v_\varphi = v_0 \sin\alpha, \quad v_r = v_0 \cos\alpha, \quad \Gamma = 2\pi R v_\varphi, \quad Q = 2\pi R h v_r$$

$$m = \frac{1}{A} = \frac{2Q}{\Gamma a} = \frac{2h}{a}\frac{1}{\operatorname{tg}\alpha}$$

Thus the parameter A is defined uniquely in terms of the angle and the apparatus dimensions.

The radius of the air vortex was measured at the distance 400 mm from the nozzle entrance by a probe introduced through a port in the end wall. The nozzle was fabricated from glass so that the instant of probe contact with the liquid surface can be clearly observed with the use of specially selected illumination.

The resulting values of y_1/a in the range of m from 0.8 to 2 for the case $l/a = 0.83$ are shown in Fig. 2 (solid circles). Also shown is curve 1, which corresponds to the maximum-discharge principle. Curve 3 is plotted using (2.8).

REFERENCES

1. G. N. Abramovich, Applied Gasdynamics [in Russian], Gostekhteoretizdat, Moscow, 1951.

2. M. A. Gol'dshtik, A. K. Leont'ev, and I. I. Paleev, "Aerodynamics of the vortex chamber," Teploénergetika, no. 2, 1961.

3. V. I. Skobelkin, "Theory and calculation of the centrifugal ejector," Tr. M-va aviats. prom-sti, no. 17, 1948.

4. L. A. Klyachko, "The theory of the centrifugal ejector," Teploenergetika, no. 3, 1962.

5. M. A. Lavrent'ev and B. V. Shabat, Methods of Complex-Variable Function Theory [in Russian], Nauka, Moscow, 1965.

6. N. N. Moiseev, "Asymptotic methods of the narrow strip type," collection: Some Problems of Mathematics and Mechanics [in Russian], Izd-vo SO AN SSSR, Novosibirsk, 1961.

24 February 1969

Novosibirsk